ANALYSIS OF INDETERMINATE STRUCTURES
BY MECHANICAL, ELECTRICAL, AND
OPTICAL MEANS

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by
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The authors wish to acknowledge their indebtedness to Professor Joseph S. Kenney of the Department of Civil Engineering who suggested the topic and acted as their advisor.

The object of this thesis is to investigate scale and micrometer methods of mechanical analysis; and, more important, to develop electrical and optical means applicable to measuring minute deflections of models.

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INTRODUCTION

More and more use is being made of the indeterminate structure. The increased use evolves from the principal motives of aesthetics and economics. Cleaner lines are made possible. For example, compare the lines of the rigid portal with its counterpart, the comparatively ugly roof truss. As for economy, this type of structure is an elastic unity; that is, the whole structure is effective in resisting a given loading, making possible the use of smaller sections. The increasing use can be seen also as a result of the development of the welded joint which is the best known method of fabricating an elastic unity. Finally, it has been realized that riveted and reinforced concrete connections in many cases are rigid or semi-rigid connections and economies can be effected by treating the structure as an indeterminate.

Even with the admitted advantages of aesthetics and economy, the indeterminate structure is not always popular. The first objection is the difficulty of theoretical analysis. A few years back the solution of such a structure demanded a fine background of mathematics; but the development of procedures and short cut methods, notably Professor Hardy Cross' method of "Moment Distribution" has in part remedied the first objection.

No longer does the design demand an extensive knowledge of mathematics; however, the developed procedures are tedious computations lending to arithmetical errors. The second objection is the uncertainty of simplifying assumptions necessary to make a theoretical solution possible. The second objection can be met by the prudent use of models. Some of the structures which lend themselves especially to analysis by models are: continuous beam, portals, arches of all kinds, steel framed buildings, and Vierendeel trusses.

There are several different methods of analysis that may be used. There is the direct method in which a loaded structure is tested for strains. This is not practical for the design of a single structure but might prove feasible where there is considerable duplication. A modification of the direct method is the construction of a scale model which is loaded, and a study is made of the strains produced. The construction of a scale model is time consuming and relatively expensive. Structures can be analyzed by measuring the slope and deflections of loaded models. This work will concern itself with the analysis of structures by the use of unloaded deflected models, the method most commonly used.

Mechanical analysis by deflected structures is based

upon Clerk Maxwell's law of reciprocal deflections, which states that the distortion, either angular or linear and measured in any given direction, of any point "a" in a beam or frame due to a unit load, either angular or linear and in any direction, at any point "b" is equal to the distortion at the second point "b" due to a unit load at the first point "a", provided that at each point the loads and distortions are always measured in the same direction.

The proof of Maxwell's theorem may be seen in the following example:

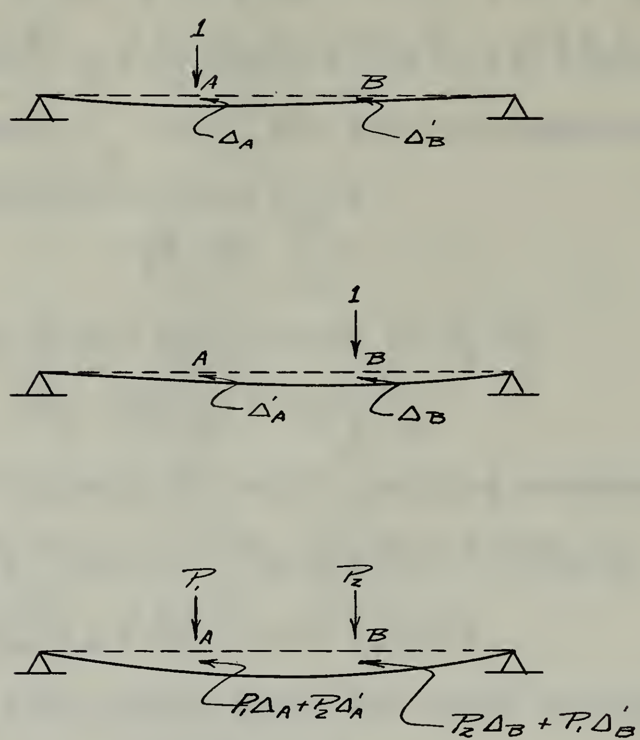


Fig. A1

In the simply supported beam shown in Fig. A1 let

Δ_A = deflection at A due to a unit load at A;

Δ'_B = deflection at B due to a unit load at A;

Δ_B = deflection at B due to a unit load at B; and

Δ'_A = deflection at A due to a unit load at B. Now, if loads P_1 and P_2 are applied to points A and B respectively, the deflection at A is $(P_1\Delta_1 + P_2\Delta'_A)$ and the deflection at B is $(P_2\Delta_B + P_1\Delta'_B)$. Considering that loads P_1 and P_2 are applied simultaneously the work done on the beam is:

$$W = \tfrac{1}{2}P_1\,(P_1\Delta_A + P_2\Delta'_A) + \tfrac{1}{2}P_2\,\Delta_B + P_1\Delta'_B)$$

$$= \tfrac{1}{2}P_1^2\Delta_A + \tfrac{1}{2}P_1P_2\Delta'_A + \tfrac{1}{2}P_2^2\Delta_B + \tfrac{1}{2}P_1P_2\Delta'_B$$

But, if the loads P_1 and P_2 are applied successively, the work done by application of P_1 is

$$\tfrac{1}{2}P_1\,(P_1\Delta_A)$$

The work added by the application of P_2 is

$$\tfrac{1}{2}P_2\,(P_2\Delta_B) + P_1P_2\Delta'_A$$

The total work done by P_1 and P_2 applied successively is

$$W = \tfrac{1}{2}P_1\,(P_1\Delta_A) + \tfrac{1}{2}P_2\,(P_2\Delta_B) + P_1P_2\Delta'_A$$

$$= \tfrac{1}{2}P_1^2\Delta_A + \tfrac{1}{2}P_2^2\Delta_B + P_1P_2\Delta'_A$$

This value of work should equal the value of work done by the simultaneous application of loads.

$$\tfrac{1}{2}P_1{}^2\Delta_A + \tfrac{1}{2}P_1P_2\Delta'_A + \tfrac{1}{2}P_2{}^2\Delta_B + \tfrac{1}{2}P_1P_2\Delta'_B$$
$$= \tfrac{1}{2}P_1{}^2\Delta_A + \tfrac{1}{2}P_2{}^2\Delta_B + P_1P_2\Delta'_A$$
$$\Delta'_B = \Delta'_A$$

Δ'_B the deflection at B due to a unit load at A is equal to Δ'_A the deflection at A due to a unit load at B.

The basis for mechanical analysis may be shown by this simple example:

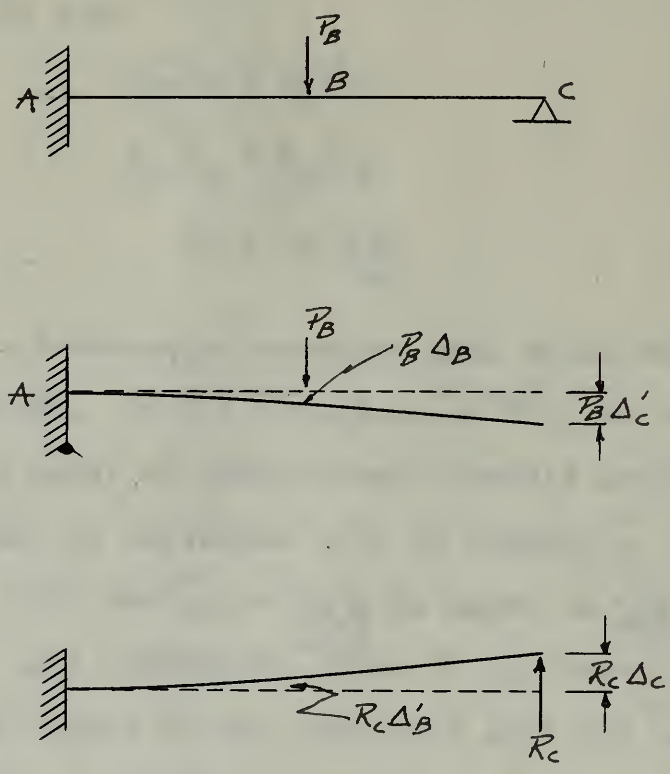


Fig. A2

If the reaction C in Fig. A2 is removed and C is allowed to deflect due to the load P_1 at B, the deflection at C is equal to $P_1\Delta'_C$ where Δ'_C is the deflection at C due to a unit load at B. Now, if the load P_1 is removed from the structure and a vertical force R_C is applied at C, the deflection at C is equal to $R_C\Delta_C$ and the deflection at B is equal to $R\Delta'_B$, where Δ_C is the deflection at C due to a unit load at C and Δ'_B is the deflection at B due to a unit load at C. For the loaded structure to be in equilibrium it is necessary that

$$R_C\Delta_C = P_1\Delta'_C$$

Since $\Delta'_B = \Delta'_C$

$$R_C\Delta_C = P_1\Delta'_B$$

$$R_C = P_B\frac{\Delta'_B}{\Delta_C}$$

This is a fundamental equation upon which mechanical analysis is based. If the structure of Fig. A2 is replaced by an unloaded model of proportional elastic qualities, and if the model is deflected at C an amount δ_C, the deflection at B will be δ_B, and $\frac{\delta_B}{\delta_C}$ is equal to $\frac{\Delta'_B}{\Delta_C}$ of the deflected real structure. Thus it is, that $\frac{\delta_B}{\delta_C}$ is the value of the ordinate of the influence line for the reaction at C, and the value of the ordinate at any other point X is $\frac{\delta_X}{\delta_C}$. Likewise, if the structure of Fig. A2 is one having a horizontal redundant and a moment redundant at C

due to the load P_1, it may be proven that

$$H_C = P_1 \frac{\delta_B}{\delta_{HC}}$$

and

$$M_C = P_1 \frac{\delta_B}{\theta_C}$$

where δ_B is the displacement of B in the direction of P_1, δ_{HC} is an applied horizontal deformation at C and θ_C is an applied rotational deformation measured in radians.

The construction of the model in most cases may be made swiftly and inexpensively. Where the structure is subjected to loads producing bending, and axial strain is negligible in comparison, the model need only represent the elastic properties of the structure. This can be accomplished by making the moments of inertia of the various cross sections proportional to the moments of inertia of the corresponding sections in the prototype. The majority of structures fall in the above category. In structures where the amount of bending is small in comparison with the strain due to axial forces, for example, a braced framework, the cross sectional areas of the model must be made proportional to the cross sectional areas of the prototype. If both bending and axial strain are appreciative, a true scale model is necessary, but fortunately, this is seldom the case.

Models may be constructed of various materials. The materials which are commonly used are: steel splines, cardboard and plastics. Each of these has advantages and disadvantages. The steel can be given large deflections, but the model is more difficult to construct, especially if it represents a structure of variable cross sections. The cardboard's chief advantage is its ease of construction, but the model cannot readily withstand wear and tear. On the other hand, a plastic model is not as easily constructed but it is capable of withstanding comparatively rugged treatment.

There are two general classifications in which analyses may be placed, depending upon whether large or small deflections are employed. Large deflections have the advantage of requiring less expensive equipment for their measurement, and the effect produced by a distortion can readily be envisioned. On the other hand, considerable error may be produced by using large deflections. The theoretical development which makes this type of analysis possible is based upon the fact that work is a function of the movement of a load point in the direction of the load. In the use of models, it is assumed that a point on the centerline moves perpendicular to the original location of the centerline.

This is not true; however, the error produced by the assumption is negligible where deflections are small, but may be considerable where deflections are large. For the purpose of the average design office the advantage of minute deflection method is offset by its main disadvantage, namely, expensive apparatus. Also, to achieve consistent results the operator must acquire a definite technique. The best known large deflection apparatus is the Gottschalk Continostat, which employs steel splines as models; and, thus far, the best known deflection apparatus is the Beggs Deformeter, which measures minute deflections.

Since the best known method of mechanical analysis, the Beggs Deformeter, is not found in the average engineering office chiefly because of the expensiveness of the equipment, it is the purpose of this work to investigate scale and micrometer methods of mechanical analysis; and, more important, to develop, if possible, electrical and optical means applicable to measuring minute deflections of models.

WITH SCALE AND MICROMETER

The Gottschalk Continostat, mentioned in the Introduction, has the disadvantage that where the structure to be analyzed has a varying cross section, the model's section must be varied with the addition of steel splines to represent the elastic properties of the structure. This may be done with some difficulty, depending upon the assortment of splines available. Also, the method of building up the model leaves much to be desired. The moment of inertia is varied in steps rather than smoothly. In certain types of structures, for example, in arches, this type of construction may cause serious errors. Although the properties of steel splines are ideally suited for large deflection apparatus, it will be shown by the following experiments that even a plastic can successfully be used as a model.

In the following experiments the plastic "Viscaloid" of uniform thickness was used. The elastic properties of the models were made proportional to the elastic properties of their prototype by making their depth at every section proportional to the cube root of the moment of inertia of their prototype at every section. This can be seen from the expression:

$$I \text{ of structure} \propto I \text{ of model}$$

$$I_S \propto \frac{bd^3}{12}$$

$$I_S \propto d^3$$

$$d \propto \sqrt[3]{I_S}$$

A model 18 inches in length, $\frac{3}{8}$ " in depth and $\frac{3}{32}$ " thick, as shown in Fig. B1, was constructed. The model was mounted on a drawing board at points A, B, and C with small finishing nails to represent a two span continuous beam supported at these points. Vertical deflection was applied at C and the influence line determined for the reaction at C. The experiment consisted of two sets of readings, one for $\frac{3}{4}$ " deflection and the other for $1\frac{17}{64}$ " deflection at C. Readings were taken at every inch along the beam and were measured from a steel bar which was clamped to the board in a position parallel to the undeflected model. A scale graduated to $\frac{1}{64}$ " was used and the deflections read to $\frac{1}{64}$ ". From the data of Fig. B1, it can be seen that with the increased deformation at C came increased accuracy.

Since a scale with the least reading of $\frac{1}{64}$ " was used, it was possible to be off a half unit in readings of the initial position and half a unit in the readings of the deflected structure giving a cumulative error of $\frac{1}{64}$ ".

Two Span Beam

Influence Line Ordinates for Reaction C.

Deflections measured with scale

	1		2		
Position	Δ_N	$\frac{\Delta_N}{\Delta_C}$	Δ_N	$\frac{\Delta_N}{\Delta_C}$	Calculated
A	0	0.000	0	0.000	0.000
1	$-\frac{1}{64}$	−.0208	$-\frac{1}{64}$	−.0123	−.0275
2	$-\frac{1}{32}$	−.0416	$-\frac{3}{64}$	−.0370	−.0523
3	$-\frac{3}{64}$	−.0624	$-\frac{1}{16}$	−.0494	−.0740
4	$-\frac{3}{64}$	−.0624	$-\frac{3}{32}$	−.0740	−.0890
5	$-\frac{3}{64}$	−.0624	$-\frac{7}{64}$	−.0865	−.0960
6	$-\frac{3}{64}$	−.0624	$-\frac{7}{64}$	−.0865	−.0925
7	$-\frac{1}{32}$	−.0416	$-\frac{3}{32}$	−.0740	−.0768
8	$-\frac{1}{64}$	−.0208	$-\frac{3}{64}$	−.0370	−.0466
B	0	0.000	0	0.000	0.000
10	$\frac{1}{16}$	.0834	$\frac{5}{64}$	.0617	.0645
11	$\frac{7}{64}$	.1460	$\frac{3}{16}$	.148	.1495
12	$\frac{13}{64}$	.271	$\frac{5}{16}$	.247	.241
13	$\frac{1}{4}$	.334	$\frac{7}{16}$	.346	.348
14	$\frac{11}{32}$	.458	$\frac{19}{32}$	.468	.466
15	$\frac{25}{64}$	.522	$\frac{47}{64}$	.580	.593
16	$\frac{35}{64}$	.728	$\frac{29}{32}$	.717	.725
17	$\frac{21}{32}$	.875	$1\frac{1}{16}$	.840	.865
C	$\frac{3}{4}$	1.000	$1\frac{17}{64}$	1.000	1.000

Δ_N = deflection at any point

$\frac{\Delta_N}{\Delta_C}$ = experimental values of ordinates

Fig B1

Furthermore, this error was just as likely at the reaction point C. Assuming that the deflection at C was not in error, a $\frac{1}{64}$" error would cause a discrepancy in the influence line ordinate of .0208" in the $\frac{3}{4}$" deflected model and .0123" in the $1\frac{17}{64}$" deflected model. The majority of the readings were within the above values.

To increase the accuracy a larger model was used together with refined measurements. The second model was 48" in length and $\frac{3}{8}$" in depth.(Fig. B2) Deflections varying from approximately $\frac{1}{2}$" to $1\frac{2}{3}$" were given to point A, and resulting deflections at every 4" point were measured with a micrometer. The micrometer used was actually a depth gage held in contact with the bar clamped to the drawing board parallel to the undeflected structure. Since friction between the board and the model could cause error, bearings were placed under the model at various points to serve as pivots. Care was also taken that the model was able to rotate freely about the nails at the reaction points A, B, and C.

From the data it may be seen that the results were close to the calculated values, but there was little difference between the results of the .502" deformation of A and the 1.67" deformation of A.

Two Span Beam

Influence Line Ordinates for Reaction A

Deflections measured with micrometer

	1		2		3		4		5		6		
Position	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Calculated
A	.502	1.000	.850	1.000	1.000	1.000	1.250	1.000	1.503	1.000	1.670	1.000	1.000
4	.396	.788	.670	.789	.795	.795	.990	.792	1.190	.792	1.322	.792	.794
8	.300	.598	.501	.590	.590	.590	.737	.590	.892	.593	.990	.592	.592
12	.206	.410	.345	.406	.407	.407	.508	.406	.611	.406	.677	.405	.407
16	.122	.243	.209	.246	.242	.242	.307	.245	.360	.239	.405	.242	.241
20	.054	.108	.097	.114	.105	.105	.135	.108	.156	.1037	.177	.106	.103
B	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
28	−.032	−.0637	−.056	−.066	−.070	−.070	−.083	−.0664	−.098	−.0652	−.112	−.0670	−.0637
32	−.050	−.0995	−.083	−.0976	−.099	−.099	−.120	−.096	−.144	−.0957	−.163	−.0975	−.0925
36	−.051	−.1014	−.081	−.0954	−.100	−.100	−.125	−.100	−.149	−.0991	−.169	−.101	−.0938
40	−.031	−.0617	−.068	−.080	−.071	−.071	−.087	−.0696	−.106	−.0705	−.128	−.0765	−.0741
44	−.022	−.0438	−.034	−.041	−.041	−.041	−.049	−.0392	−.058	−.0386	−.067	−.040	−.0406
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Δ_N = deflection at any point

Δ_A = deflection at reaction A

$\frac{\Delta_N}{\Delta_A}$ = experimental values of ordinates

Fig. B2

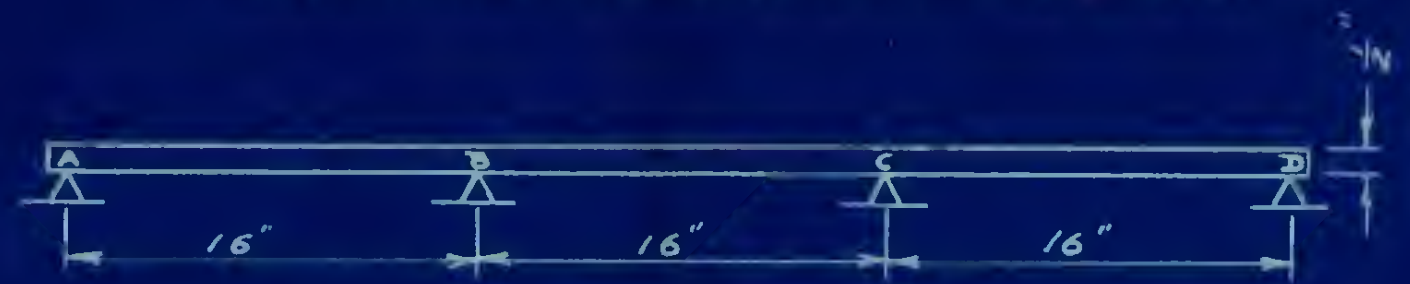
This, of course, was due to the increased accuracy of the micrometer method over the scale method. Knowing when the micrometer came into contact with the model was the greatest difficulty encountered. Since the model was of plastic material it had the tendency to creep under the applied deformation, but, as far as was discerned, the creep had no effect on the results. The model acted as an elastic body with the application of the deformation at A and took the shape of the influence line for reaction A. Immediately after the application of the deformation, the model began to creep, that is, it had the tendency to take the shape of the deformed structure as a permanent set and therefore had no maleffect on results. The way that subsequent deformations were applied is of interest. After each set of readings, the model was not brought back to its zero position, but the deflection at A was increased to the value desired for the next set of readings, thus illustrating that creep had no effect on the results.

Another experiment was performed using the second model except that the model was mounted at points A, B, C, and D as shown in Fig. B3 to represent a three span continuous beam. Deflection was again applied at point A and the procedure of the preceding two span analysis followed.

THREE SPAN BEAM

Influence Line Ordinates for the Reaction of A

Deflections measured with micrometer



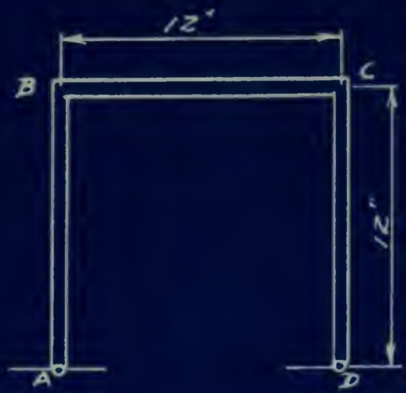
	1		2		3		4		
Position	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Δ_N	$\frac{\Delta_N}{\Delta_A}$	Calculated
A	.233	1.000	.478	1.000	.773	1.000	1.021	1.000	1.000
4	.154	.662	.322	.675	.513	.664	.692	.676	.687
8	.088	.378	.185	.397	.300	.388	.403	.394	.394
12	.038	.163	.075	.157	.124	.160	.166	.162	.162
B	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	-.017	-.073	-.035	-.073	-.050	-.065	-.067	-.065	-.071
24	-.014	-.060	-.032	-.067	-.049	-.063	-.069	-.067	-.075
28	-.012	-.0515	-.021	-.044	-.030	-.039	-.041	-.040	-.040
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
36	.004	.017	.015	.031	.017	.022	.022	.0215	.022
40	.003	.013	.019	.040	.021	.027	.012	.012	.025
44	.020	.086	.016	.033	.008	.010	.005	.005	.017
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Δ_N = Deflection at any point

Δ_A = Deflection of reaction A

$\frac{\Delta_N}{\Delta_A}$ = Experimental values of ordinates

FIG. B3



RIGID PORTAL

Influence Line Ordinates for Horizontal Reaction at D

Deflections measured with micrometer

Position	1		2		Calculated
	Δ_N	$\frac{\Delta_N}{\Delta_D}$	Δ_N	$\frac{\Delta_N}{\Delta_D}$	
A	0.000	0.000	0.000	0.000	0.000
2	.033	.108	.046	.107	110
4	.057	187	.087	.203	195
6	.095	.311	.129	.301	.288
8	.120	.394	165	.385	.370
10	146	474	.199	.464	.442
B	.168	.550	.227	.525	.500
14	020	.0655	.032	0746	0416
16	.023	.0754	.030	.070	.0666
18	.031	.101	.043	.100	.075
20	.030	.098	.036	.084	.0666
22	.013	.042	.017	.0396	.0416
C	.160	.525	.215	.502	.500
26	.172	.564	.235	.548	.558
28	.197	.645	.263	.614	.630
30	.215	.705	.298	.695	.712
32	.244	.800	.338	.789	.805
34	.275	.902	.383	.894	.900
D	.305	1.000	.429	1.000	1.000

Δ_N = Deflection at any point

Δ_D = Horizontal deflection at D

$\frac{\Delta_N}{\Delta_D}$ = Experimental values of ordinates

FIG. B4

A model of a pin supported rigid portal was next constructed with dimensions as shown in Fig. B4. A deflection of .305" horizontal deflection was given to point D and the influence line for the horizontal reaction of D was determined. All measurements were again made with a micrometer. A second horizontal deflection of .429" was applied to D and readings taken. The results were satisfactory.

It was noted in the several mechanical analyses that the largest percentage errors occurred at the points of minimum deflection, but these were the points where loading produced the least effect upon the redundant under consideration.

Although continuous beams and a rigid portal of constant section were considered, good results might also be obtained for continuous beams and rigid portals of varying section, for arches of constant and varying sections, for braced arches, sewers, etc., provided that reasonable care is taken in selecting the scale of the model, and in taking the measurements. The construction of a model for a braced arch is somewhat different from the construction of continuous beams and rigid portal models. Assuming that the forces on all bars are axial, the majority of strain energy is utilized in shortening or lengthening the bars rather than in producing bending; therefore, the cross section of the bars of the model must be made proportional to the cross sections of the bars of the prototype.

USE OF

RESISTANCE, INDUCTANCE AND CAPACITANCE

AT

AUDIO FREQUENCY

Although good results may be obtained by the large deflection methods of analysis, they do not compare with the results obtainable with the Beggs Deformeter. The Deformeter's success lies in the fact that minute deflections are employed, which is more in keeping with what happens in the actual structure, and that the method employs an excellent microscope capable of consistently reading deformations to one ten thousandths of an inch. Any other method of mechanical analysis which would give results comparable to that achieved with the Deformeter would necessarily have a means of measuring minute displacements comparable with the excellent microscope. An answer to the problem might be the conversion of minute displacements to more easily measured quantities. Displacements could be made to work changes in the resistance, inductance or capacitance of an electrical circuit.

<u>Resistance for Displacement Conversion</u>

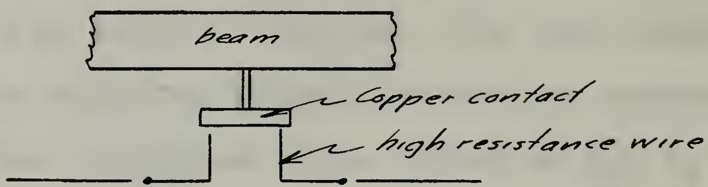


Fig. C1

A small copper contact was attached to the model so that when the model deflected the contact moved over a section of high resistance nichrome wire (Fig. C1), increasing or decreasing the length of wire in the circuit. Since the resistivity of nichrome was known, it was attempted to calculate displacement by measuring the changes in resistance effected by the moving contact.

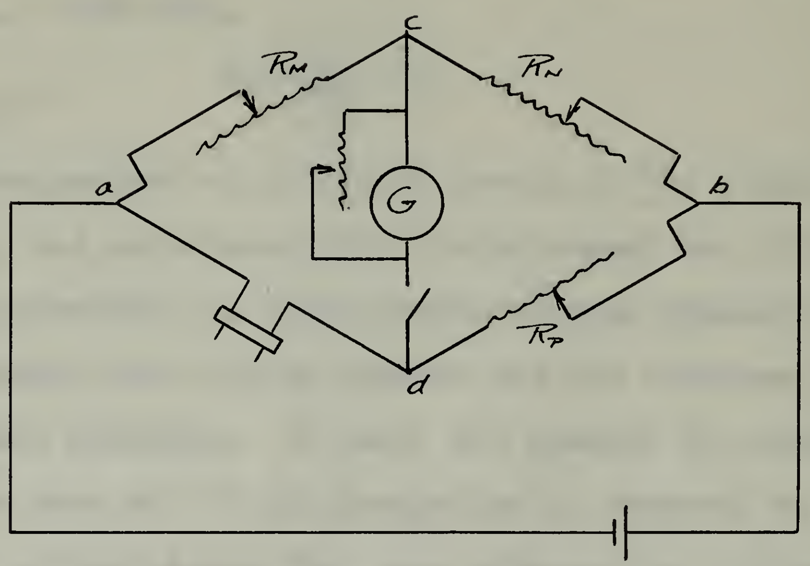


Fig. C2

Resistance measurements were made by means of a Wheatstone Bridge as shown in Fig. C2. The high resistance nichrome wire with the sliding contact was connected in as R_x and decade resistance boxes served as R_M, R_N and R_P. A 6 volt battery was used as the current source.

A sensitive type of wall galvanometer, manufactured by Leeds and Northrup Company, was used to balance the bridge. When the bridge was balanced there was no difference of potential between "C" and "d".

$$I_M R_M = I_X R_X$$

$$I_N R_N = I_P R_P$$

Since no current flowed into the galvanometer, $I_M = I_N$ and $I_X = I_P$. Therefore,

$$R_X = \frac{R_M}{R_N} R_P$$

The galvanometer was sensitive enough to the slightest current, and resistance could be measured; but it could not be correlated to displacement because contact resistance between the sliding contact and the nichrome wire did not remain constant. In fact, the changes in contact resistance were out of all proportion as compared to the changes of resistance that were effected by displacement.

Inductance for Displacement Conversion

An impedance bridge as shown in Fig. C3 was constructed and used in this experiment for determining the desired inductances. An audio oscillator was used to

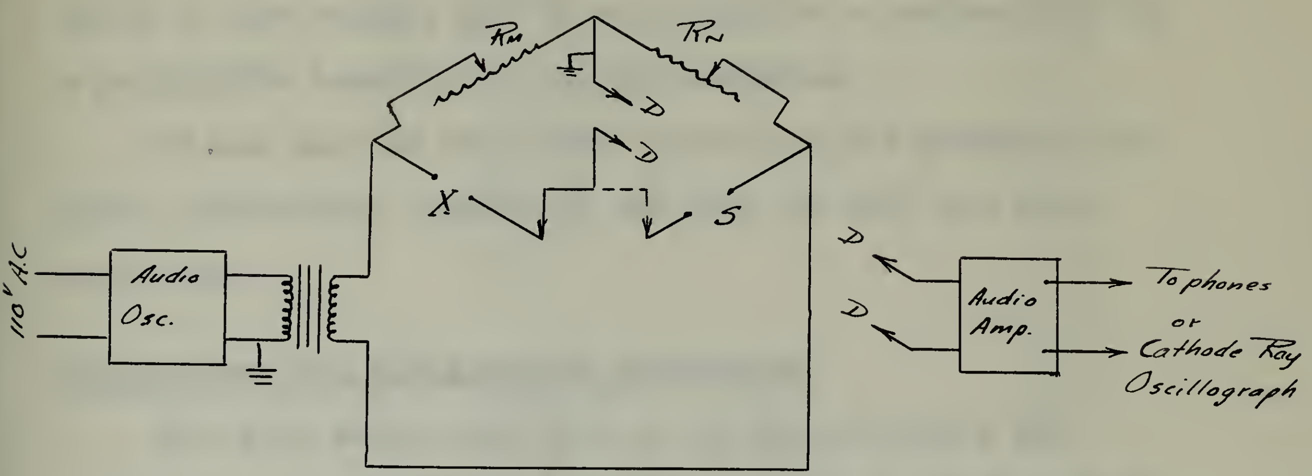


Fig. C3

increase the frequency so that the bridge could more easily be balanced by means of the cathode-ray oscillograph or by means of phones. It was connected into the bridge through a 1 to 1 transformer in order that the oscillator would actually be isolated from the bridge. Otherwise, changing the bridge constants might change the frequency of the oscillator.

Various sizes of coils were placed in the arm X and in the arm S. It was hoped that in deflecting, a model

might be able to move a soft iron core in or out of a coil so as to change the inductance of the coil, and thereby, serve to measure the amount of displacement. However, it was found that only with comparatively large coils and cores was it possible to produce measurable effects. Also the results were affected by mutual inductance in the various parts of the bridge; but it was possible to reduce this to a negligible quantity by proper shielding.

It was decided that this system was not practical at audio frequencies because of the size of coil and core necessary.

Capacitance for Displacement Conversion

The same set-up was used as in the preceding experiment except that the inductive reactance of the arm X was replaced by a capacitive reactance and the arm S by a standard capacitor. In this experiment it was proposed that minute displacements in the capacitor plates in arm X would cause changes in capacitance which could be measured with the aid of the oscillograph. This experiment was unsuccessful at audio frequencies with practical sizes of plates.

In using the impedance bridge and Wheatstone bridge, the standard arms were checked to see if they were as calibrated, for the precision of the results depends not only upon the sensitivity of the balance but also upon the precision of the calibration of the standard arms.

USE OF

INDUCTANCE AND CAPACITANCE

AT

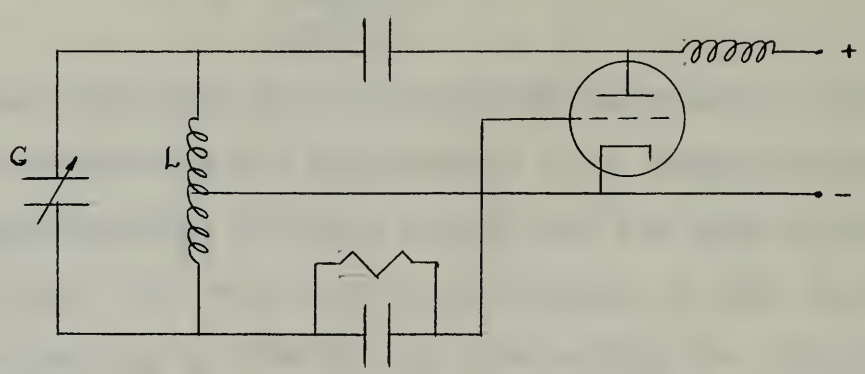
RADIO FREQUENCY

Electronic Oscillator Principles

One of the basic principles that has found wide and varied use in all branches of industrial development is that employed in the electronic oscillator. The purpose of this section will be to investigate the suitability of this device in measuring the minute deflections required in the analysis of structures by the use of unloaded scale models.

Before suggesting an application of this device to the problem with which this thesis is concerned, it would be well to discuss, at least in general terms, the principle upon which it functions, so that the difficulties involved may be better understood and overcome.

Most oscillator circuits are made up essentially of a parallel resonant circuit, sometimes called a tank circuit because of its ability to store energy, and an amplifying tube connected as in Fig. D1.



This is a simple electron tube oscillator of the Hartley type, one of the basic oscillator circuits. In the above circuit, as in many of the most commonly used oscillator circuits, the resonant frequency is dependent on the value of inductance and capacitance in a tuned circuit connected to the grid and plate circuits of an electron tube. At resonance, in parallel connected tuned circuits, the impedance is a maximum and, therefore the current flowing is a minimum. As for any parallel resonant circuit, with resistance in both branches, the frequency of resonance in this case may be calculated by the formula

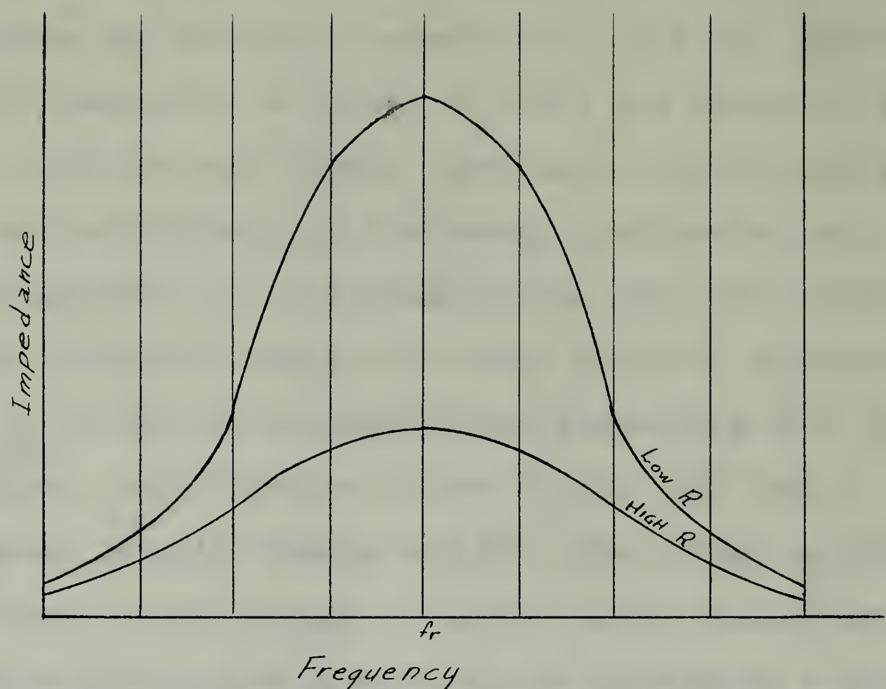
$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{CR_L^2 - L}{CR_C^2 - L}}$$

However, the calculations are greatly simplified if the branch resistances are neglected, in which case the above equation reduces to

$$f_r = \frac{1}{2\pi\sqrt{LC}} \times 10^6$$

where the frequency is in kilocycles per second, inductance is in microhenries and capacitance is in micro-microfarods. This approximation is close enough for the vast majority gradio work. The value of the resistance in the tuned circuit is important, however, in determining the sharpness of

the resonance curve, since that indicates the sensitivity of the circuit. A very sharp resonant curve, which results from a low value of resistance in the branch circuits, has a well defined peak value of equivalent impedance in the tuned circuit when operating at resonant conditions, as indicated by curves in Fig. D2.

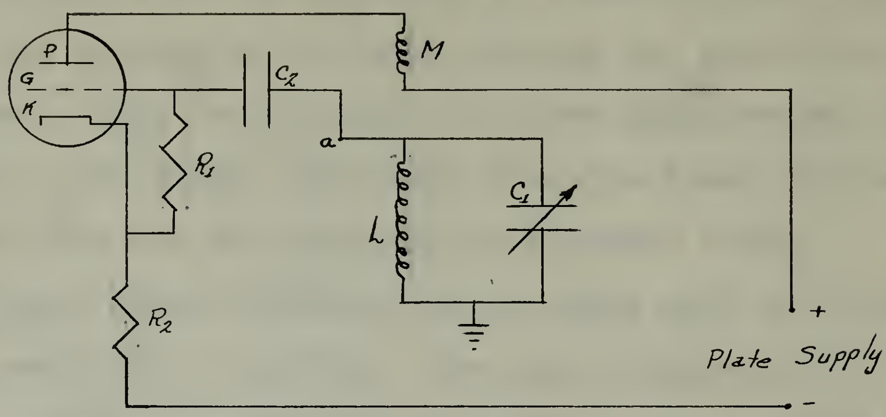


The sharpness of a circuit is referred to as the quality or Q of a circuit and is usually defined as the ratio of total inductance to total resistance. Since the inductive reactance is usually the controlling value, the Q is in most cases given as the ratio of inductive reactance to the resistance in the inductive branch. A high value of Q indicates that a circuit is lightly damped, has a sharp resonant peak, and high selectivity. Q for radio circuits

is usually in the range of 50 to 200 and sometimes up to 500. Audio circuits use values of from 1 to 20.

The operation of an oscillator circuit has often been explained by using the analogy of a loaded spring or of a pendulum. In all three instances, the high energy level in one system is transferred to a second system of lower energy level, which in turn transfers the energy to the first system and the cycle repeats until all the original energy is dissipated in losses or until the energy of both systems is at the same level. Oscillatory motion may be maintained indefinitely if the energy lost during each cycle is supplied by a suitable source. The most common method of returning energy to a tuned parallel resonant circuit is to utilize the amplifying properties of a thermionic tube. Amplification refers to the fact that a small change in grid voltage has the same effect on plate current that a large change in plate supply voltage has. This fact indicates the possibility of generating a large voltage by a small one, or the generation of a large amount of power from a small amount. The amplified power or voltage output from the tubes is obtained from the emf. connected between the plate and cathode, not from the tube itself. The tube merely regulates the power from an outside source. This method is employed in the tuned-grid

circuit shown in Fig. D3.



The fluctuation of potential at point "a" due to oscillations in the tuned circuit cause variations in the grid voltage, "G". The change in plate current resulting from this causes a transfer of energy from the plate voltage supply to the tuned circuit inductance through the mutual inductance between the two coils. If the circuit constants are properly chosen, the rate of energy fed back to the circuit will be equal to the rate of energy lost due to resistance. Similar systems have been developed for other oscillator circuits.

A very necessary part of any oscillator circuit is a well regulated power supply. Since the characteristics of the power supply to the oscillator affect to a very marked degree the stability of the oscillator, it would be well to discuss that subject briefly.

The practically universal use of alternating current for transmission and distribution of electric power necessitates the conversion to direct current for certain applications. This transformation is done quite readily by electronic rectifiers, especially when the direct current output is used for the operation of electron tubes.

A simple diode electron tube is often used as a rectifier in small radio circuits. When the cathode of the tube is heated, it emits electrons which are attracted to the plate, if the plate is at a positive potential with respect to the cathode. Because of the nature of thermionic conduction, the tube is a conductor only in one direction. If an alternating voltage is applied between the cathode and the plate, then electrons will flow only on the positive half-cycles of alternating voltage. There will be no flow of electrons during the half-cycle when the plate is negative with respect to the cathode. Thus the alternating current is rectified to a pulsating direct current which is suitable, with proper filtering, for use in electron tube operation. The direct current required for their operation may also be obtained from a battery source.

Practically all radio-frequency oscillators are essentially tuned amplifier circuits with means provided whereby some of the output is fed back into the input,

the amplifier supplying its own signal.

Because of the requirements for the amplifying process, the tube in oscillators employing a tank circuit must be connected to the circuit in such a way that when the alternating grid-to-cathode voltage is positive, the alternating plate-to-cathode is negative.

A large number of circuits have been developed and their relative merits and particular uses are discussed very well in several books on the subject, including "Electronic Circuits and Tubes" by the Cruft Electronics Staff of Harvard University, and "Theory and Applications of Electron Tubes" by H. J. Reich. The beat-frequency oscillator will be discussed later in the section on instrumentation.

A satisfactory oscillator for laboratory use must satisfy the requirement that the frequency of oscillation is stable; that is, the frequency does not drift from the desired value. This stability is affected by the temperature of the working circuit, the value of plate voltage, the load that is supplied by the output, and mechanical variations of circuit elements. Temperature changes result in a change of all three circuit parameters and cause a certain amount of drift. This may be minimized by using the equipment only after a sufficient warm-up period to

bring the circuit to the operating temperature and by exercising careful temperature control during the measuring operation.

Another factor that must be considered in the design of these circuits is the problem of spurious capacitance and inductance such as may occur in leads and connectors. This difficulty may be minimized by proper shielding and judicious arrangement of the component parts of the circuit.

The moisture content of the air has some effect on the capacitance between two plates where air is the dielectric and should be considered when precise measurements are desired.

Proposed Applications

The most obvious application of oscillators to the problem of converting linear displacements into measureable electrical quantities is to use a simple oscillator in which the resonant frequency of the tank circuit may be changed by varying the capacitance. A deviation from any "zero" resonant value caused by a displacement of capacitor plates attached to the model will cause a change of current in the plate circuit and will also change the resonant frequency. This output current or frequency can be detected in any of several ways and with proper calibration may be interpreted in terms of distance.

Since the capacitance between two plates varies inversely with the distance between them, some method for returning to "zero" conditions at the beginning of each measurement must be devised. This may be done by using a particular frequency or current as the "zero" value. This value will, of course, correspond to some known displacement and can be duplicated when desired. Then under ideal conditions a change in distance between the plates results in a corresponding variance from zero conditions in other parts of the circuit. One thing to bear in mind in this system is that an increase in capacitance does not have the same effect upon frequency as does a decrease in capacitance. The change of frequency is given by the equation:

$$\Delta f_r = \frac{1}{2\pi\sqrt{LC}} - \frac{1}{2\pi\sqrt{L(C+\Delta C)}} = \frac{1}{2\pi\sqrt{L}}\left(\frac{1}{\sqrt{C}} - \frac{1}{\sqrt{C+\Delta C}}\right)$$

This indicates that an increase in capacitance has less effect in changing the frequency than does a decrease. Therefore, two calibration curves must be plotted; or the displacement motion must be limited to one direction. The sensitivity of this arrangement, that is, the change in frequency per unit change of displacement, depends on the magnitude of the original distance between plates, due to the fact that capacity varies inversely with distance. In other words, the smaller the zero displacement, the more effective will be a unit change of distance in causing a change of frequency.

Some of the controlling features of the physical construction should be considered. The frequency range desired during operation is the controlling factor of design and determines the constants of the circuit. There are any number of combinations of inductance and capacitance that will give a desired frequency, since the frequency depends only on the product of the two quantities. As previously mentioned, the Q of the circuit should be high, and further the ratio of inductance to capacitance should be as low as possible. In this method, where the change in frequency depends on a change of capacitance, the total capacity of the circuit should be small so that

a small change will be a larger percentage of the total. Preliminary investigations showed that frequencies in the neighborhood of 1 or 2 megacycles give good sinsitivity, but spurious capacitance gives instability and makes duplication of results very difficult. A frequency somewhat lower, around 500 kc., was required to give a suitable balance of sensitivity and stability.

A change of 10 or 20 kc. is noticeable and may be detected quite easily on a frequency meter or on a well calibrated radio set. Assuming two capacitor plates that move perpendicular to the planes in which they lie, and with air as the dielectric, plates with an area of 2.25 square inches would require a travel of 0.45 inches (from 0.05 to 0.50 inch) to give a change of 10 mmfd. Larger plates and smaller initial displacements would, of course, require smaller travels to give the same change. However, the least dimension measured accurately would be about 0.10 inch, depending on the scale of the frequency meter or the radio dial. This would be two large for use in this problem. Difficulties in getting the plates back to the zero position before each reading would be very severe. Also keeping the plates in perfect alignment as they move apart would require precise machining of parts. The supports would probably give friction causing an additional load to be applied to the deflected structure, though this

effect could be minimized by proper lubrication.

The suitability of this system for solving the problem seems doubtful, but greater attention to details could reduce the inaccuracies to a minimum. Higher frequencies, calling for lower total capacitance would keep the size of the capacitor plates within practical dimensions as well as requiring smaller displacement to give noticeable change in frequency. The use of amplifying equipment on the output end of the oscillator would give a finer indicating scale. The sensitivity and the difficulty in duplicating results would seem to make the system unsuitable.

A second method that presents itself is similar to the first except that a change in inductance of the tank circuit is used rather than a change of capacitance. The physical set-up is quite the same except that a small coil with a soft-iron core is used as the conversion unit. Displacements are registered by the change in relative position of the coil and core. Physically, one of the two parts(coil or core) is attached to the model while the other is secured to a base plane. A change of position of the model with respect to the base plane causes a change in inductance and consequently a different resonant frequency.

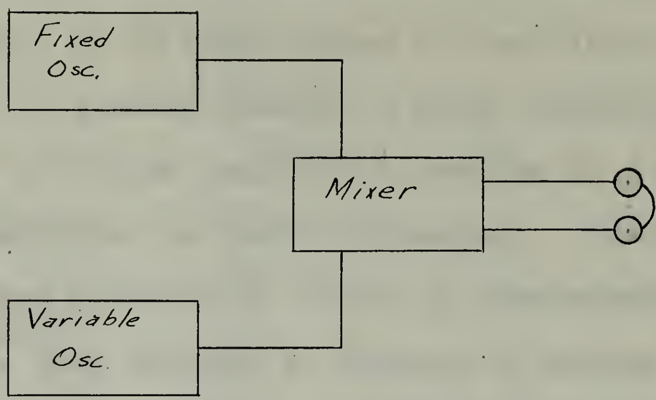
A small coil with a soft-iron core, placed in parallel or series with a standard radio oscillator coil may be made to serve as the conversion unit.

The physical characteristics of the coil are not of great importance except that the coils should be as small as possible and yet large enough to be effective in changing the inductance of the circuit. For stability, the total value of inductance should be high to give a high value of Q, but not too high to give a reasonably low value of $\frac{L}{C}$. Actually the capacity may be changed, within limits, to give the desired frequency for any set of coils that may be connected in the tank circuit. These capacitors are usually called trimmer condensers and are used extensively in radio work to control the resonating frequency of tank circuits in both receiving and transmitting units. They are usually multi-plated with mica as the dielectric.

High sensitivity is attained more easily in this type of circuit since the permeability and consequently the inductance changes very rapidly with respect to distance at the instant the core is completely withdrawn from the coil. At this point the high permeability path changes abruptly into one of low permeability since no iron remains in the path. The range of high sensitivity is very limited, extending over less than one-quarter of an inch in one coil that was tested.

An apparatus that may be used to give better accuracy in indicating the change in frequency due to a change of circuit constants in the tank circuit is called the beat-

frequency oscillator. This device consists essentially of two separate oscillators that feed into a common mixing circuit, with the output of the mixer indicating the difference between the two input frequencies. A simple block diagram is shown in Fig. D4.



As an example, if one oscillator resonates at 700 kc. and the other at 600 kc., the output from the mixer will be 100 kc. If the difference in frequencies is zero there will be no output from the mixer. This point may be used as the indicating or "zero" position.

Any of the usual oscillator circuits may be used for this purpose, the only requirement being that both oscillators should be as nearly alike as possible.

The design of these beat-frequency or heterodyne oscillators involves a number of special problems, the most important of which is the prevention of interaction between the two oscillators. This effect causes the two oscillators

to pull into synchronism when their output frequency difference is small. Interaction may be prevented by adequate shielding, proper location of component parts, and use of filters in connecting the oscillators to the mixer circuit.

Frequency instability in heterodyne units results from the same causes as in other types of oscillators, but it is likely to be greater because a small percentage change in frequency of either oscillator results in a much greater percentage variation of output frequency. However, if the oscillators are practically alike in arrangement and value of constants, they respond to changes of supply voltage and temperature in a similar manner and the output frequency is affected very little.

As to practical design values for a circuit, it is necessary to decide on the desired operating range of frequency and consequently determine the values of inductance and capacitance required in the tank circuits of the oscillators. At higher frequencies, above 2 mc, the problem of interaction is very pronounced, and heavy shielding of connectors plus isolation of parts has to be resorted to. If the situation warrants, extensive filtering systems and buffer amplifiers are used to prevent distortion and drift of the output voltage frequency.

Either the inductance or the capacitance of the circuits may be used as the conversion unit. As in the other methods, a system of "zero" conditions would be required. Both tank circuits must be controlled so that "zero" conditions may be arrived at easily and with accuracy.

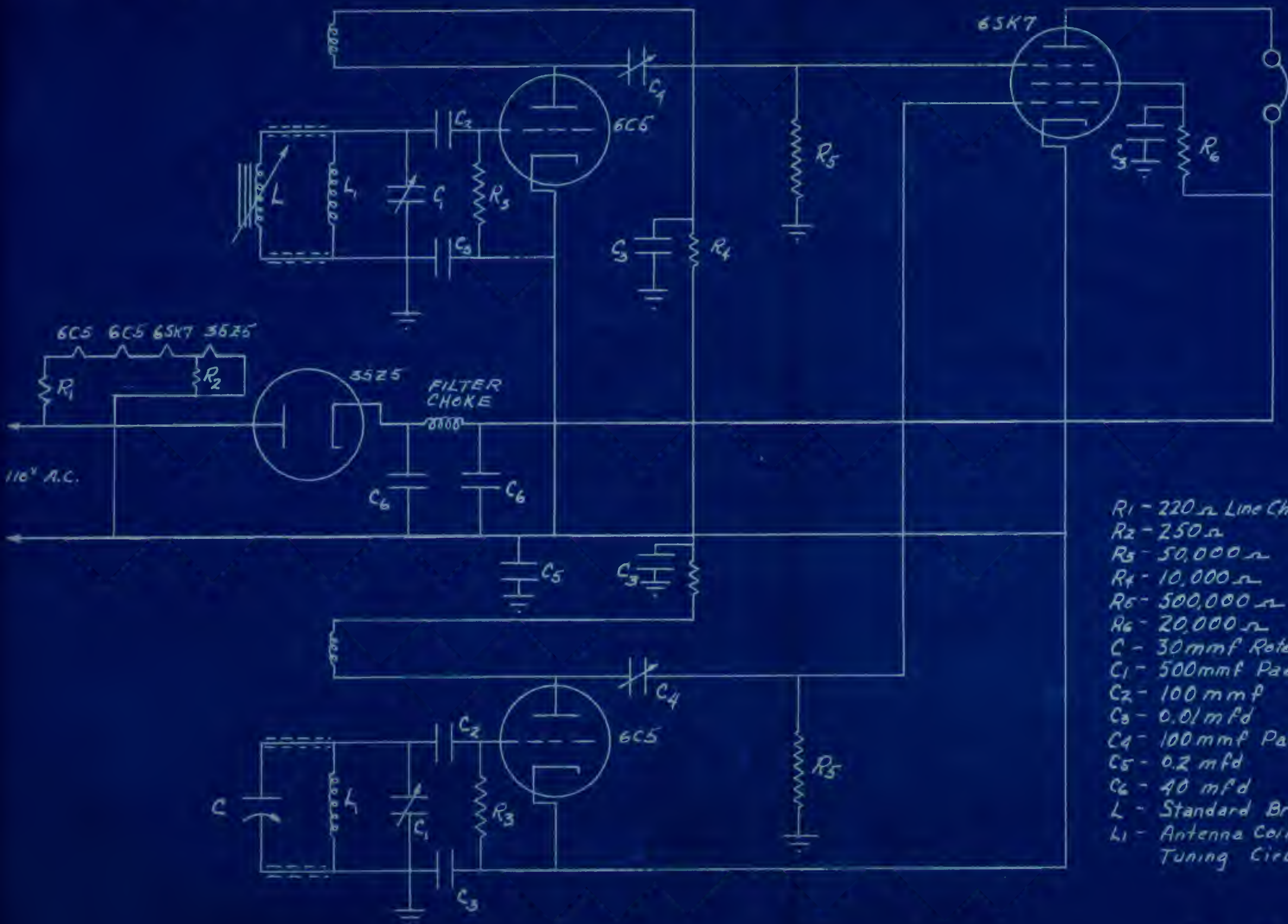
The theory of application of the heterodyne unit is quite obvious--one oscillator is considered as "fixed" in the sense that it is the one used as a standard, while the tank circuit of the second oscillator is attached to the model and its frequency of oscillation is changed by merit of the relative motion of two capacitor plates or of a coil and an iron core.

In the former case, one plate of the condenser in the variable oscillator circuit is attached to the model, the other to a base surface. The condenser in the fixed oscillator might best be of the rotor type used in commercial radio tuning circuits. This would give relatively easy calibration since the rotor condenser could be returned to its zero setting and the variable condenser plates could be adjusted until a zero beat is obtained from the mixer. Then a movement of the condenser plates, caused by deflection of the model, causes a change of frequency in the variable oscillator circuit. This is immediately reflected in the output of the mixer by the presence of a beat note in a set of earphones. This beat note may also be picked

up by a radio set tuned to the frequency of the oscillators. The problem then resolves itself into one of getting rid of the beat by changing the capacity of the "fixed" condenser in accordance with the change that has already occurred in the variable condenser. If the fixed condenser has been previously calibrated, the deflection of the two plates may be read directly from a scale that is attached to the rotor condenser or from a calibration curve that has been set up.

The main coils of the oscillator circuits may be operated on in the same manner as has been suggested above for the condensers. The ease of tuning the "fixed" circuit is not as great when inductance coils are used as compared to condensers, since the critical range in both oscillators is so narrow. On the other hand, however, the instrumentation in regard to attaching the converter unit to the model is somewhat simplified by using the coil. The physical size of the coil is much smaller than the size of condenser plates to give the same sensitivity. From the standpoints of practicality and ease of tuning, it would seem better to make the conversion unit inductive and the balancing and measuring unit capacitive, the latter using a rotor condenser.

The last suggested method has been carried through as far as design and construction are concerned, and



R_1 - 220 Ω Line Chord Resistor
R_2 - 250 Ω
R_3 - 50,000 Ω
R_4 - 10,000 Ω
R_5 - 500,000 Ω
R_6 - 20,000 Ω
C - 30 mmf Rotor Condenser
C_1 - 500 mmf Padder Condenser
C_2 - 100 mmf
C_3 - 0.01 mfd
C_4 - 100 mmf Padder Condenser
C_5 - 0.2 mfd
C_6 - 40 mfd
L - Standard Broadcast Coil
L_1 - Antenna Coil from Permeability Tuning Circuit

FIG. D8

preliminary investigations have been made into its accuracy and suitability.

The circuit used is indicated in Fig. D8. The oscillator circuits are tuned so as to resonate at approximately 650 kc. Preliminary tests to determine the total range that can be covered indicated that it is slightly less than one-quarter of an inch when operating in the most sensitive region of the coil. The coil used in this investigation was the antenna coil taken from a permeability tuning circuit used for small radios. The balancing condenser was composed of a single plate of a constant wave-length condenser, giving a maximum capacity of 30 mmfd.

The results obtained were very erratic due primarily to high friction between the model and the base board. Also, the instrument used had the tuning condenser mounted on a separate chassis from the oscillator resulting in the use of long leads between the two units. This caused quite a bit of interference from stray capacity and inductance as can be readily demonstrated merely by bringing the hand to within about an inch of the connector. Placing of the tuning condenser on the same chassis with the oscillators would permit the enclosure of the connectors within the chassis and would greatly reduce the interference from stray sources.

Another difficulty resulted from making the shielding

can nuts serve as a ground for several wires. Any force causing a strain in these nuts affected the resonating frequency causing it to drift quite badly. This indicates that greater care must be exercised in placing grounds so that they will be more stable.

In regard to the connector between the oscillator and the conversion coil, it would seem better to have a single lead(with heavier shielding than ordinary phonograph pick-up wire)between the oscillator and the conversion coil. This single lead could be used with several coils if a switching arrangement were worked out so that only one coil is connected at a time.

One very serious difficulty that arises is the need for having a number of coils of the same inductive properties. These coils would have to be constructed with very close tolerances so that mutually consistent results may be obtained from each of them when several are used on a single model.

Further experimental work is required to determine the effect of length of circuit from coil to oscillator on the results. Also, the amount of variation in coil properties is not at present available.

It was planned to calibrate the machine by moving (from a zero position) the core in the inductive circuit a known distance as measured by a micrometer and ascertaining

the amount of rotation required by the rotor condenser to bring the circuits back into balance as indicated by the absence of a beat note in the earphones. Curves could than be plotted showing the rotation from zero position versus the linear displacement of the core with respect to the coil. The ability to duplicate this curve would indicate the stability and the suitability of the instrument.

A series of curves obtained as already indicated, except at different frequencies and ratios of $\frac{L}{C}$(in either or both oscillator circuits), would indicate the point where optimum operating conditions (considering sensitivity, stability, and range) could be expected.

This instrument, when properly adjusted, would seem to give very excellent results. A better method of calibrating would give greater accuracy, though the inherent instability of the machine may not warrant such an extreme.

OPTICAL METHOD

The optical method presents another approach to the problem of converting minute displacements of a scale model to a more easily measured quantity. When a model is displaced from its initial position it assumes a deflected position and every point on the model assumes a definite slope. Mirrors mounted at any point, on the model, would also assume the slope of the deflected model, and in so doing are forced to rotate through some angle, equal to the slope at the point of mounting. Therefore, a reflected ray of light from an incident ray would also be rotated, its angular rotation being equal to twice the angular rotation of the mirror. The following proof is offered.

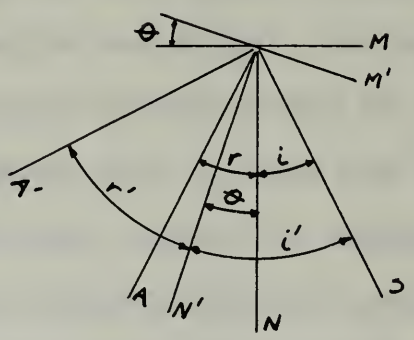


Fig. 1

If an incident ray of light from a light source S, Fig. 1, falls upon a plane mirror M at an angle i with the normal N; it is reflected at an angle r. The angle of incidence i equals the angle of reflection r. Rotating the mirror to position M', through an angle θ, causes the normal to rotate through the same angle to position N', and the reflected ray along A'. The angle of incidence increases to $i' = i + \theta$ and similarly $r' = r + \theta$. The angle of rotation of the reflected ray:

$$AOA' = r' - N'OA = r' - (r - \theta)$$

Since $\quad r' = i',\ r = i,$ and $i' = i + \theta$:

$$AOA' = i' - (i - \theta) = (i + \theta) - (i - \theta) = 2\theta$$

Thus the rotation of the reflected ray is twice the rotation of the mirror. It is with this fact in mind that the particular problem, to convert minute linear displacements in terms of more easily measureabel quantities, i.e., tangent distances and angles, was approached.

For a proper understanding of the problem involved, let it be assumed that a model has been constructed with very small mirrors mounted at designated points. The model may be of any material (usually a plastic compound), size, or shape, constructed to represent the actual structure for which the investigation is to be conducted. For the sake of simplicity in this discussion, a two span continuous beam supported at points A, B, and C, Fig. 2 will be considered.

A single line A,B,C, will be used to represent the model diagrammatically. However, the discussion will be applicable to any shaped model to be investigated.

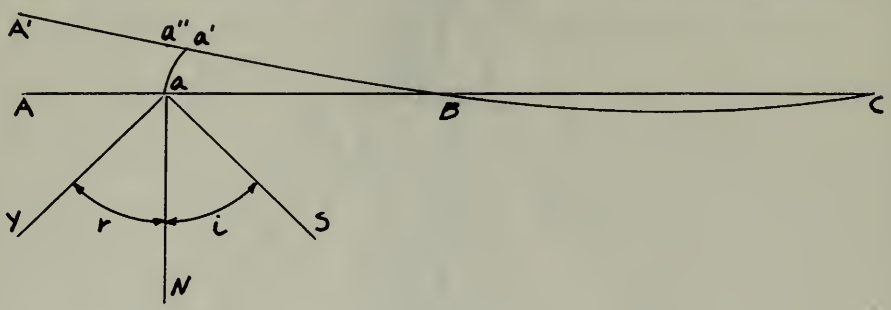


Fig. 2

Take the model resting on some flat surface and in the initial unrestrained position A,B,C, fig. 2. If a ray of light from a light source S falls upon the plane mirror, attached to the model at any point A, at some angle i to the normal; it is reflected at an angle r from the normal. From the previous discussion i and r are equal. Displace point A a small amount to point A' and the continuous beam will be forced into the deflected shape shown in Fig. 2. In so doing, the mirror moves from point a to a' along some curved path aa', rotating thru some angle. The problem would be simplified immeasureably if the mirror did move along the straight line path aa".

Consider the portion of the beam at points a and a', Fig. 3. In moving from a to a', the mirror, assumed to act as an integral part of the model, rotates through the angle $\mathcal{L}$.

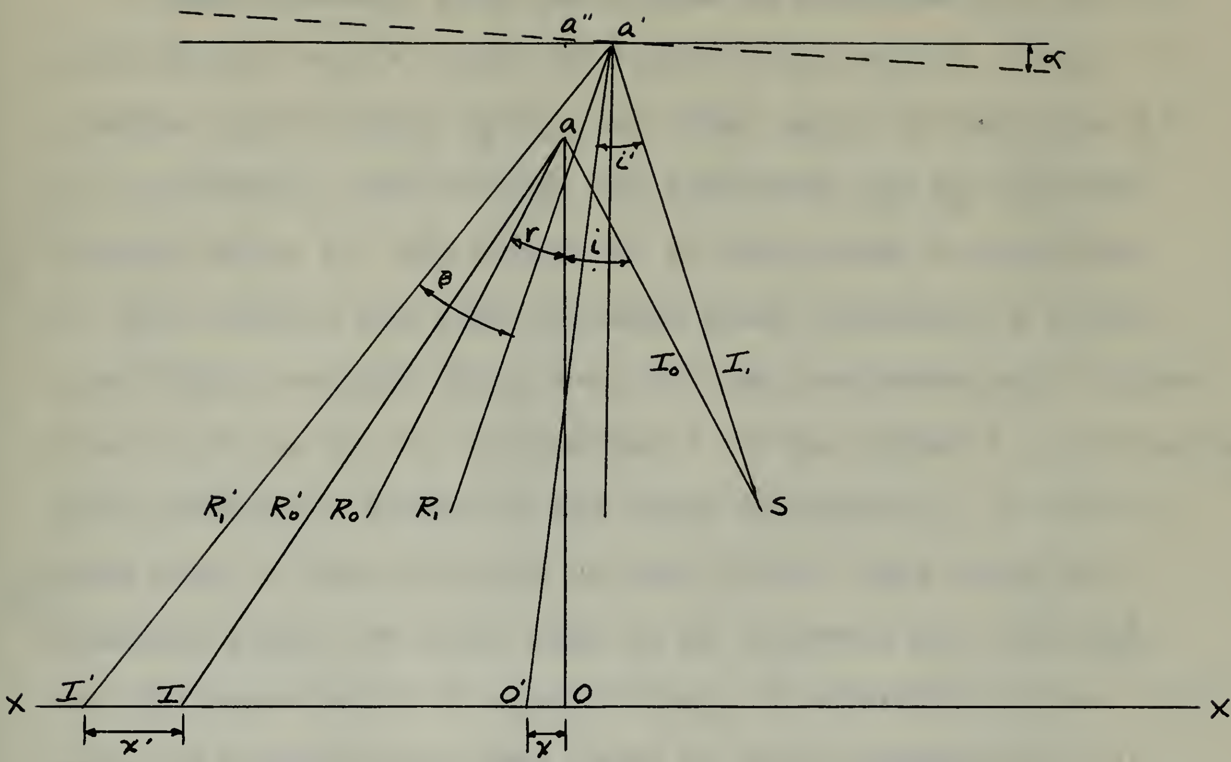


Fig. 3

A tangent to the deflected model at this point, extended, intersects line AB; and subtends an angle equal to the angle α which gives the slope of the deflected model at this point. Measurement of the angles through which the mirrors rotate would then give the slopes of the deflected curves at these points.

Since at the present no known mathematical solution is available to compute the values of these angles a method of measurement was adopted. Before proceeding with this method a basic assumption must be made: The error introduced in angle by the mirror in moving from a to a' is negligible.

Theoretically, with the mirror in position a', Fig. 3, the incident ray I_o would no longer fall upon it. But, another ray of light I_1 at some other angle of incidence i' to the normal, does so with the reflected ray R_1' rotated through angle B. The normal N' to the mirror in position a' also rotates and when extended would intersect a graduated scale, mounted along x-x, at some predetermined distance from A, B, C, at o'. The normal N to the mirror in position a, when extended, intersects the scale at point o. If point o were used as the reference or zero point, from which all readings along the scale were to be measured and referred to, the error would be equal to x. In addition there would be another error x', equal to the difference in readings of rays R_o and R_1'; both rays reflected from the rotated mirror with the same angle setting, in positions a and a', respectively. The errors may be additive or compensating. Position of the light source may be important and no tests were carried on to determine the most desireable position for it. The light source, approximately 18 inches from the model, was considered relatively close enough to assume, for all practical purposes, these errors introduced would be negligible. The equipment was then constructed and arranged with this assumption in mind. Before continuing, it is best to note that all preliminary

tests to determine the feasibility of the optical method were to be conducted on a continuous beam before proceeding to models of other shapes. As such, the following procedure will apply to a continuous beam and may, with some slight modification, if necessary, be applied to other models.

A simple experiment was conducted to determine the order of magnitude of these errors with the aid of the calibration instrument. Its construction and primary use will be explained later. With a hypothetical curve drawn between points a and a', point a' 2" from and 2/10" to the right of point a; center the calibration instrument over point a with the axis of the mirror coinciding with the longitudinal centerline of the base B, and line AB, with the indicating pointer reading zero. With light falling on the mirror an image of the etched cross hair on the face of the mirror will appear on the graduated tape at x-x. This reading will serve as the zero reading, and should be taken with the light source perpendicular to the plane of the mirror. Readings should be taken for several angle settings of the mirror at any point a, with the light source remaining perpendicular and fixed in position. An approximate range of angles of 20 degrees, 10 degrees either side of the zero point should suffice; because the maximum angle of rotation to be expected of the mirrors attached to the model, shown by preliminary investigations and observations, would in all probability fall within this range of angles. Similarly, the instrument

was then moved to point 2, located $\frac{1}{2}$" from and 1/10 " to the right of a, and finally to a'. For each angle setting on the mirror readings were taken and at all times the longitudinal center line of the base of the instrument was parallel to the line AB. Readings recorded are tabulated below and do show very little change in initial readings.

Position	a	2	a'
Angle			
0°	5.1	5.15	5.09
5°	6.1	6.12	6.12
10°	7.21	7.22	7.19

The tabulated results appear to be conclusive, but they should not be interpreted as being wholly so, at least not until further investigation is carried on after the necessary refinements in the construction of the apparatus, graduated tape, and protractor are made. This test was conducted for an extremely hypothetical curved path, but from observations in displacing point A approximately 1", which is considered large, the path was for all practical purposes a straight line. It can be assumed that these errors are negligible and therefore angle O'a'I' = OaI. Thus within probable accuracy, tangent distance readings on the graduated scale at x-x can be correlated to the corresponding measured

angles of rotation of the calibration mirror; for initial and final positions of designated points on the model. The procedure would then be to measure the angle through which the calibration mirror rotates, record the graduated scale reading and tabulate these values. With the known scale readings for the deflected model enter the table and select the corresponding angle.

Apparatus

The entire assembly, photograph # 1, consists of; a work bench B, telescopic adjuster unit TE, telescopic support S, transformer T, model M_o with attached mirrors M, rail R, adjusting screws S', and calibration instrument(not shown).

Work Bench

The work bench top was made of 3/4 " plywood having sufficient strength and rigidity to remain perfectly flat. The edges were made mutually perpendicular so that they could be used for alignment with the wall, riding rail R, and telescopic unit TE. One inch angle irons extending 6" from the edge of the 3/4 " plywood served as supporting arms for the traveling rail. The angles, secured to the end legs, held the rail upon which the telescopic adjuster unit traveled. Each arm contained adjusting screws, to permit adjustment and leveling of the rail. A $\frac{1}{4}$" square

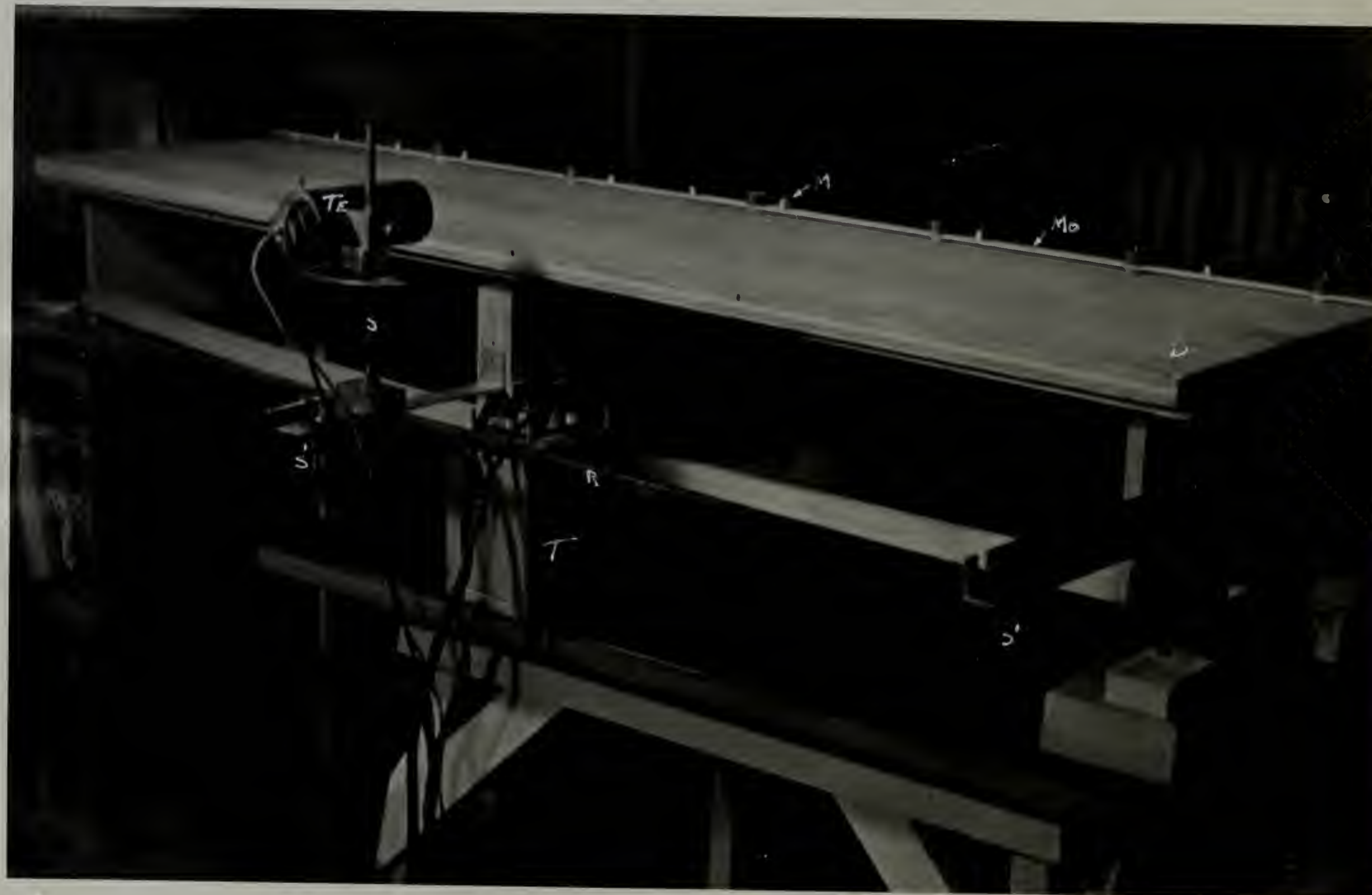


PHOTO #1

PHOTO #2

steel rod was used for the rail. Strips of angle iron, approximately $\frac{1}{2}$" in length, were attached to the ends of the end legs with wood screws. Each angle leg was drilled and tapped to receive screws to aid in adjusting and leveling the work bench.

<u>Telescopic Unit</u>

The telescopic unit, photo # 3, consists of a lens and two pieces of tubing; one piece sliding within the other

PHOTO.# 3

to permit focusing of the light source. The lens was set into the stationary piece of brass tubing S, with the use of an adapter ring. This ring was carefully machined to give a very snug fit. A grooved recess in the ring held a wire spring, extending over the entire periphery, and held the lens flush against a shoulder, which was provided. This adapter unit was an offspring of an original idea to use very small mirrors manufactured by General Electric, which required the use of a lens. It proved to be unsatisfactory and was discarded in favor of another ; a brass block B, silver soldered to tube S, comtains a drilled hole which permits upward and downward movement and horizontal rotation of the telescopic unit. Protractor P_R, was used to measure the horizontal movement of the light source. Adequate light of sufficient intensity was provided by a Tung-Sol bulb, rated at 6-8 volts producing 50 candle power. This type of lamp is normally found in truck head lights. A wood plug W, inserted in the adjustable tube section contained a drilled hole of sufficient diameter to permit insertion of the base of the bulb. An overlapping brass plug B_R, 1/8 " thick, was fastened by two wood screws to the exposed face of the wood plug. A light brass compression spring was inserted between the head of a brass screw passing through a drilled hole in the center of the plug and the inner face of the plug and served to insure

proper contact. A brass nut served to insure proper adjustment of contact.Lead wires were soldered to the lamp and to the proper terminals of the transformer TR. A $\frac{1}{4}$" round rod R_o, used for a support, was attached to a base block A, machine grooved to permit travel along the riding rail R_o. A set screw C, in the side of the block was used to fasten the support in place.

Transformer

A filament transformer was used to reduce house voltage, 110 volts AC, to the required voltage for the lamp, 6-8 volts AC.

Calibration Instrument

The original intent of this instrument was to provide a means for measuring the angles through which a mirror rotates. For each angle a scale reading was to be recorded and these values tabulated. With known readings for the deflected position enter the tables and select the proper angle. The instrument, photograph # 4, consists of a calibration mirror M, which should contain an etched hair line on the surface; support S, grooved to receive the mirror; base B, containing a forced fit spindle about which the mirror and support were centered; pointer I, and protractor P. used to record the angles.

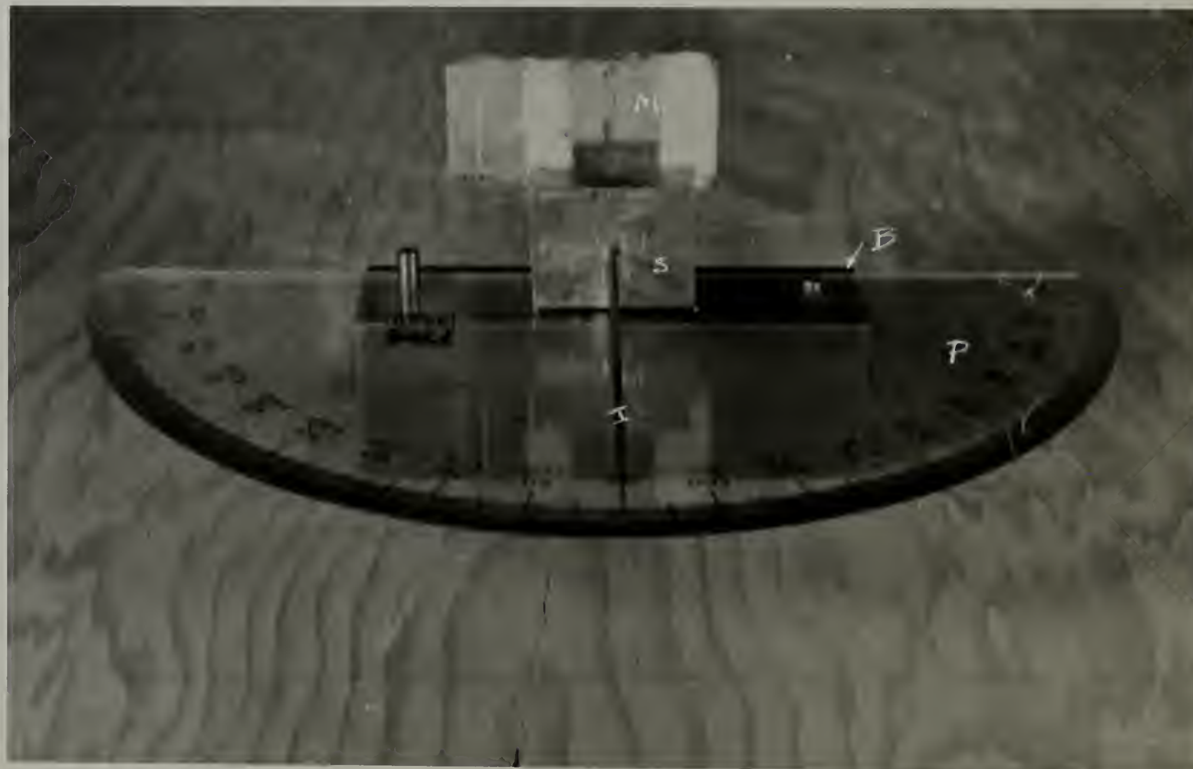


PHOTO #4

Mirrors

Ease in handling, mounting, and size were of paramount importance in selecting mirrors. A mirror small enough to be considered a point on the model was desired. For a preliminary trial .015" x .090" mirrors manufactured by General Electric, NP113142, were used. These were attached to the edge of the model with special mirror glue, also produced by the General Electric company. Wet tip tooth-picks served to remove them from their individual capsules, and then they were mounted on the prepared edge. The glue requires approximately 10 hours to set. The model had to be handled with considerable care once the mirrors were mounted. The slightest jarring action would cause them to fall off.

A double convex lens of 5 cm. focal length, mounted in the adapter unit, was required to concentrate the light source at a point on the mirror. A circular image was produced containing fringes around the periphery; which may possibly be a result of light interference or defraction. Glue on the mirror surface reduced its reflecting properties. Because of the difficulties involved in the above method, the following method was attempted, then adopted.

Small mirrors cut from ordinary hand mirrors served their purpose well. The smallest strips possible with an ordinary glass cutter were cut from the piece of stock and

then cut in pieces approximately 5/16 " long. The face of each small mirror was covered with adhesive tape. A narrow strip of the tape was removed with a razor and straight edge, exposing only that portion of the reflecting surface. A satisfactory clear image was obtained with method. A suggested modification would be to eliminate the adhesive tape entirely and etch hair lines on the face of the mirrors to give a mark of fine division. Another suggestion, which may prove just as satisfactory, would be to scribe a fine line on the back face of the mirror with a scriber and straight edge. Airplane glue was used to attach the mirrors to the model satisfactorily. It has great strength and drys quickly.

Proposed Procedure

The procedure in mind at the time would be to set the edge of the table at a known measured distance and parallel to the wall of the room; with the measuring tape mounted on this wall. Level the table, rail, and set the model at a measured distance from the table edge. With the model in position, either trace the outline of the model on the table and transfer the selected points and set the calibration instrument at each one of these points; or place the instrument over the points on the model directly. With the instrument at each point and light source perpendicular to the face of

the calibration mirror, locate the zero point reading. Then calibrate the angles of rotation of the mirror with the graduated scale on the wall. Tabulate these values of the angles and tangent distances for the unrestrained position of the model. For models with straight edges parallel to the wall one calibration need be necessary and for models of other shapes all points will probably have to be calibrated. The pin removed, displace the model a known amount, and replace the pin. With the scale readings for the deflected model enter the tables and select the corresponding angles. The angle between the normal and reflected ray equals twice the angle of rotation of the mirror. With the value of this angle and the tangent distance OI', Fig. 3, known, the perpendicular distance Oa" can be computed. The difference between Oa" and the measured value Oa would give the displacement aa" of point a. The ratio of aa' and the measured applied deflection, when plotted, will give the influence line.

Results and Recommendations

The method does lend itself to possible use and may prove practical, but before this can be definitely established some modifications and necessary refinements in the apparatus must be made. These refinements should include a scale graduated to at least 1/100 of an inch; at least a fifteen minute protractor on the calibration instrument; and

just as important, finer workmanship in construction, preparation, and arrangement of the apparatus. The element of time would not permit these revisions to be made, and although tests could have been conducted to obtain data the results would have been erroneous. Therefore, no data was obtained but observations from preliminary investigation and experimentation did show the need for the above suggested revisions and refinements. Application of the values of the angles measured by this method in the formulas derived for the Slope Deflection Method may have possibilities. Further investigation is necessary.

CONCLUSIONS

Mechanical methods of analysis employing large deflection of models may be used to advantage by the average engineering office. The large deflections employed makes unnecessary the use of expensive apparatus necessary when measuring minute deflections. Deflections can be such that their measurement with a finely divided scale or, better yet, with a micrometer will in many cases give results sufficiently accurate to check theoretical solutions. The results are largely dependent upon the use of good judgement in the selection of model dimensions and upon the amount of distortion applied at the redundant.

A plastic model may be used even with large deflections. The model will creep with any but the smallest deflection; however, the creep will produce no maleffect on results since creep tends to take the shape of the deflected structure as a permanent set.

The use of changes in resistance, inductance, or capacitance at audio frequencies to measure minute deflections proves to be impractical. Deflections may be made to work changes in resistance, inductance, or capacitance. However, to measure the changes and correlate them to minute deflections it is necessary that, in the case of resistance, contact resistance remain constant; in the case of inductance or capacitance impractical sizes of coil and core or capacitor plates would be necessary. If practical sizes

of inductive coils or capacitors are used, it may be possible to correlate changes to deflection but exceedingly expensive apparatus would be needed.

Radio frequency oscillators may be used in the solution of the problem provided proper balance is maintained between sensitivity and stability of oscillation, both of which depend on the circuit constants. Practical difficulties limit the application of a purely capacitive or inductive circuit connected to a single oscillator. The beat frequency oscillator reduces the problem to that of balancing one oscillator against another and thereby removes the human factor almost completely. A good method of calibration is needed in order that the results may be more reliable. It is recommended that further experimentation be carried out to determine the optimum operating conditions with the present instrumentation.

Further refinements are required in the optical method before suitable results may be obtained.

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